Theorem 4.2. *If $X_1, X_2, \ldots$ are independent and identically distributed random variables, if $g(x, \theta)$ is continuous over $\mathscr{X} \times \Theta$ where $\mathscr{X}$ is the range of X_1 and Θ is a closed and bounded set, and if $\mathscr{E} \sup_{\theta \in \Theta} |g(X_1, \theta)| < \infty$, then*

$$\lim_{n \to \infty} \sup_{\theta \in \Theta} \left| \frac{1}{n} \sum_{i=1}^{n} g(X_i, \theta) - \mathscr{E} g(X_1, \theta) \right| = 0 \quad a.s.$$

Moreover, $\mathscr{E} g(X_1, \theta)$ is a continuous function of θ.