$$\mathscr{E}\bar{X}_n = \mathscr{E}\frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{n}\sum_{i=1}^{n}\mathscr{E}X_i = \frac{1}{n}\sum_{i=1}^{n}\mu = \mu$$

$$\begin{aligned}\operatorname{Var}(\bar{X}_n) &= \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\operatorname{Cov}(X_i, X_j) = \frac{1}{n^2}\sum_{i=1}^{n}\operatorname{Cov}(X_i, X_i)\\ &= \frac{1}{n^2}\sum_{i=1}^{n}\operatorname{Var}(X_i) = \frac{\sigma^2}{n}\end{aligned}$$

$$\begin{aligned}\mathscr{E}S_n^2 &= \mathscr{E}\left\{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2\right\}\\ &= \mathscr{E}\left\{\frac{1}{n-1}\left[\sum_{i=1}^{n}X_i^2 - n(\bar{X}_n)^2\right]\right\}\\ &= \frac{1}{n-1}\left[\sum_{i=1}^{n}\mathscr{E}X_i^2 - n\mathscr{E}(\bar{X}_n)^2\right]\\ &= \frac{1}{n-1}\left[\sum_{i=1}^{n}(\operatorname{Var}X_i + \mu^2) - n(\operatorname{Var}\bar{X}_n + \mu^2)\right]\\ &= \frac{1}{n-1}\left[n(\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right] = \sigma^2\end{aligned}$$

$$\begin{aligned}M_{\bar{X}}(t) &= \mathscr{E}e^{t(X_1+\cdots+X_n)/n} = \mathscr{E}\prod_{i=1}^{n}e^{(t/n)X_i} = \prod_{i=1}^{n}\mathscr{E}e^{(t/n)X_i}\\ &= \left[M_X\left(\frac{t}{n}\right)\right]^n\end{aligned}$$

$$\mathscr{E}\left[\frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma}\right] = \frac{\sqrt{n}}{\sigma}\mathscr{E}(\bar{X}_n - \mu) = 0$$

$$\operatorname{Var}\left[\frac{\sqrt{n}\,(\bar{X}_n - \mu)}{\sigma}\right] = \frac{n}{\sigma^2}\operatorname{Var}(\bar{X}_n) = 1$$

$$\begin{aligned}M_{\sqrt{n}(\bar{X}-\mu)/\sigma}(t) &= \mathscr{E}e^{\sqrt{n}t(X_1+\cdots+X_n)/(n\sigma) - \sqrt{n}t\mu/\sigma}\\ &= e^{-\sqrt{n}t\mu/\sigma}\left[M_X\left(\frac{t}{\sqrt{n}\sigma}\right)\right]^n.\end{aligned}$$