There are versions of the monotone and dominated convergence theorems that apply to conditional expectation.

Theorem 2.7 (Monotone Convergence Theorem). *If* $0 \leq Y_n(\omega) \leq Y_{n+1}(\omega)$ *and* $\mathscr{E}|Y_n| < \infty$ *for* $n = 1, 2, \ldots,$ *then*

$$\lim_{n\to\infty} \mathscr{E}(Y_n|\mathscr{F}_0)(\omega) = \mathscr{E}\left(\lim_{n\to\infty} Y_n \,\middle|\, \mathscr{F}_0\right)(\omega)$$

except for ω *in some event* E *with* $P(E) = 0$.

Theorem 2.8 (Dominated Convergence Theorem). *If* $|Y_n(\omega)| \leq Z(\omega)$ *for* $n = 1, 2, \ldots,$ $\mathscr{E}|Z| < \infty$, *and* $\lim_{n\to\infty} Y_n(\omega) = Y(\omega)$ *except for* $\omega \in F$ *with* $P(F) = 0$, *then*

$$\lim_{n\to\infty} \mathscr{E}(Y_n|\mathscr{F}_0)(\omega) = \mathscr{E}(Y|\mathscr{F}_0)(\omega)$$

except for ω *in some event* E *with* $P(E) = 0$.