Theorem 2.4. *If both $\mathscr{E}|Y|$ and $\mathscr{E}|Z|$ are finite, then*

$$\mathscr{E}(aZ + bY \mid \mathscr{F}_0) = a\mathscr{E}(Z|\mathscr{F}_0) + b\mathscr{E}(Y|\mathscr{F}_0).$$

If, in addition, $Z(\omega) \leq Y(\omega)$, then $\mathscr{E}(Z|\mathscr{F}_0) \leq \mathscr{E}(Y|\mathscr{F}_0)$. Because conditional expectation is not uniquely defined, these relationships may not hold for ω in some set E that has $P(E) = 0$.

Theorem 2.5 (Law of Iterated Expectations). *If $\mathscr{E}|Y|$ is finite, $\mathscr{F}_0$ and $\mathscr{F}_1$ are sub-σ-algebras of $\mathscr{F}$, and $\mathscr{F}_0 \subset \mathscr{F}_1$, then*

$$\mathscr{E}(Y|\mathscr{F}_0)(\omega) = \mathscr{E}[\mathscr{E}(Y|\mathscr{F}_1)|\mathscr{F}_0](\omega),$$

except on an event E with $P(E) = 0$. In particular, $\mathscr{E}Y = \mathscr{E}[\mathscr{E}(Y|\mathscr{F}_0)]$.

Theorem 2.6. *If Z is $\mathscr{F}_0$-measurable and both $\mathscr{E}|Y|$ and $\mathscr{E}|YZ|$ are finite, then*

$$\mathscr{E}(ZY|\mathscr{F}_0) = Z\mathscr{E}(Y|\mathscr{F}_0).$$

In particular, $\mathscr{E}(Z|\mathscr{F}_0) = Z$.