Two important convergence results that apply to expectation are the following.

**Theorem 2.2** (Monotone Convergence Theorem). *If* $0 \leq f_n(\omega) \leq f_{n+1}(\omega)$ *for* $n = 1, 2, \ldots$, *then*

$$\lim_{n \to \infty} \int f_n(\omega) \, dP(\omega) = \int \lim_{n \to \infty} f_n(\omega) \, dP(\omega).$$

**Theorem 2.3** (Dominated Convergence Theorem). *If* $|f_n(\omega)| \leq g(\omega)$ *for* $n = 1, 2, \ldots$, $\int g(\omega) \, dP(\omega) < \infty$, *and* $\lim_{n \to \infty} f_n(\omega) = f(\omega)$ *except for* $\omega \in E$ *with* $P(E) = 0$, *then*

$$\lim_{n \to \infty} \int f_n(\omega) \, dP(\omega) = \int f(\omega) \, dP(\omega).$$