

The integral on  $(\Omega, \mathcal{F}, P)$  thus defined satisfies the standard properties of an integral, such as

1.  $f(\omega) \leq g(\omega) \Rightarrow \int f(\omega) dP(\omega) \leq \int g(\omega) dP(\omega)$ .
2.  $f(\omega) \leq g(\omega)$  and  $P\{\omega: f(\omega) \neq g(\omega)\} > 0 \Rightarrow \int f(\omega) dP(\omega) < \int g(\omega) dP(\omega)$ .
3.  $\int af(\omega) + bg(\omega) dP(\omega) = a \int f(\omega) dP(\omega) + b \int g(\omega) dP(\omega)$ .
4.  $A, B \in \mathcal{F}$  and  $A \cap B = \emptyset \Rightarrow \int_{A \cup B} f(\omega) dP(\omega) = \int_A f(\omega) dP(\omega) + \int_B f(\omega) dP(\omega)$ .