1. For a nonnegative random variable $X$, that is, a random variable with $X(\omega) \geq 0$ for all $\omega \in \Omega$, define

$$\mathcal{E}X = \sup\{\mathcal{E}X_N : X_N(\omega) = \sum_{i=1}^{N} x_i I_{F_i}(\omega),\ 0 \leq X_N(\omega) \leq X(\omega)\}.$$

There are no restrictions on the choice of disjoint $F_i$ other than membership in $\mathcal{F}$. Either the set

$$\{\mathcal{E}X_N : X_N(\omega) = \sum_{i=1}^{N} x_i I_{F_i}(\omega),\ 0 \leq X_N(\omega) \leq X(\omega)\}$$

has an upper bound, in which case $\mathcal{E}X$ is a finite, nonnegative number, or it has no upper bound, in which case $\mathcal{E}X = \infty$.

2. Define the expectation of a nonpositive random variable $X$, that is, a random variable with $X(\omega) \leq 0$ for all $\omega \in \Omega$, to be $-\mathcal{E}(-X)$.

3. A random variable can be decomposed into its nonnegative part $X^+ = XI_{[0,\infty)}(X)$ and nonpositive part $X^- = XI_{(-\infty,0]}(X)$; they satisfy

$$X(\omega) = X^+(\omega) + X^-(\omega).$$

If either $\mathcal{E}X^+ \neq \infty$ or $\mathcal{E}X^- \neq -\infty$ then, define

$$\mathcal{E}X = \mathcal{E}X^+ + \mathcal{E}X^-.$$

Otherwise, $\mathcal{E}X$ is left undefined.