



Figure 1. A Bivariate Random Variable Defined on the Double Coin Toss Probability Space. The sample space is $\Omega = (0, 1) \times (0, 1)$, shown in the upper right panel on which is defined the random variable $(X, Y)(\omega_1, \omega_2) = (\omega_1 + \omega_2, \omega_1 - \omega_2)$. Its range $R = (X, Y)(\Omega)$ is shown in the upper left panel.

1. (25%) Shown in the upper panels of Figure 1 is a plot of the domain $\Omega = (0, 1) \times (0, 1)$ and range $R = (X, Y)(\Omega)$ for the random variables

$$X = \omega_1 + \omega_2$$

$$Y = \omega_1 - \omega_2$$

Find the density $f_{X,Y}(x, y)$ of (X, Y) . Find the inverse image of the set A shown in the lower left panel of Figure 1 and draw it in the lower right panel. Compute the probability of A .