DEFINITION 1.1 A probability space is the triple $(\Omega, \mathcal{F}, P)$ consisting of a sample space Ω, a σ-algebra $\mathcal{F}$ of subsets of Ω, and a function P defined on $\mathcal{F}$ that satisfies the axioms of probability:

1. $P(A) \geq 0$ for all $A \in \mathcal{F}$.
2. $P(\Omega) = 1$.
3. If $A_1, A_2, \ldots \in \mathcal{F}$ are disjoint, then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$.

PROPOSITION 1.1 Let $(\Omega, \mathcal{F}, P)$ be a probability space and let A, B, and $A_1, A_2, \ldots$ be sets in $\mathcal{F}$. Then

1. $P(\emptyset) = 0$.
2. $P(A) \leq 1$.
3. $P(A) + P(\tilde{A}) = 1$.
4. $P(A \cap B) + P(A \cap \tilde{B}) = P(A)$.
5. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
6. If $A \subset B$, then $P(A) \leq P(B)$.
7. If A_1, A_2, $\cdots$ are mutually exclusive and exhaustive, then $P(A) = \sum_{i=1}^{\infty} P(A \cap A_i)$.
8. $P(\bigcup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i)$ (countable subadditivity).