



**Figure 1.5.** Illustration of Proposition 1.1. The sample space is  $\Omega = (0, 1] \times (0, 1]$ . The area of a set  $A$  is equal to its probability  $P(A)$ . The upper left panel shows  $P(A \cap B) + P(A \cap \bar{B}) = P(A)$ . The upper right panel shows  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . The lower two panels show that  $\bigcup_{i=1}^5 A_i = \bigcup_{i=1}^5 A_i^*$  where  $A_1^* = A_1$  and  $A_i^* = A_i \cap [\sim (\bigcup_{j=1}^{i-1} A_j)]$ . Therefore,  $P(\bigcup_{i=1}^5 A_i) = P(\bigcup_{i=1}^5 A_i^*) = \sum_{i=1}^5 P(A_i^*) \leq \sum_{i=1}^5 P(A_i)$ .